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Computer Technologies In School Geometry Education

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Abstract

This article was based on the author's reflections on the new ways of teaching geometry in school associated with computer technologies. We face the need for radical changes to the traditional way of teaching geometry. We came to the understanding of this fact from analysing the new trends in geometry research and the new demands on mathematics education in the present state of the art. The traditional way of teaching geometry in Russian schools which is based on Euclid's "Origins" has failed to keep up with the new demands. The future progress in geometry education will depend greatly on international cooperation and sharing of new experiences in the teaching of the subject. In this paper I shall focus on some experiences which are associated with the problems of computer analysis and control of the pupils' activities while solving geometrical problems and with the geometry training of advanced pupils.

Introduction

Is it necessary to study geometry in school? The answer to this question is obvious. Yes, it is. Let us ask one more question: why is it necessary to do it? Before answering this important question, let us consider the principal goals of teaching mathematics in school. By studying the history of mathematics, psychology, philosophy and the works of the great mathematicians of the past ages, I came to the conclusion that the principal goals of teaching mathematics in school should be universal in nature and should contribute in making an individual more knowledgeable and "intelligent". The goals are as follows:

- inculcate logical thinking and space imagination;
- know mathematical modelling and understand its role in nature and science;

- understand and be able to apply new methods of keeping, processing and transmitting information;
- develop algorithmical thinking and acquire skills of using computer for solving mathematical problems.

As we can see, these goals are not numerous but they are very important. From them we can derive the role and the functions of geometry in school education. From our point of view, geometry serves as a connecting link between school mathematics and the environment. Having established itself in ancient times as a science in space forms and relationships between them, geometry must remain closely so in the framework of a school course. The role of geometry in the evolution of space imaginations and logical thinking is well known.

However, the teaching experience on geometry in Russia and some other countries indicates that many pupils have difficulties in studying geometry and consider geometry as their "unloved" subject. This is unfortunate. In our opinion the reason first of all lies with the traditional way of teaching geometry based on Euclid's "Origins". Its characteristic features are abstractness of the geometrical concepts and clearly defined axiomatic approach. The separation of school geometry into two entities of plane and space corresponds to Euclid's "Origins" as well. So this traditional system did not correspond to the principal goals of teaching mathematics in school. The solution to this problem is in working out new ways of teaching geometry and the new principles of the school geometry course. We have devised a new system of teaching geometry in school which is different from the traditional one and which is based on the principal goals of teaching mathematics. In our system a special role has been given to computer technologies.

The Main Principles of School Geometry Teaching

Our system of teaching geometry in school is based on the following two principles:

- visualization;
- "I am in space".

In contrast to the traditional system we actually do not use axiomatic approach. We obtain geometrical facts (theorems on the properties of the figures) not by means of unclear and abstract reasoning but by using geometrical models of an environment. Logical strength of our reasoning has been achieved not by using axioms but by the hidden applications of the fundamental geometrical concepts such as a set, convex hull, transformation and some others. Let us consider in more detail the principles mentioned above.

These principles are linked firmly and cannot be separated. First of all we study the main geometrical facts about simple space polygons (cube, prism, pyramid) and some others (ball, cylinder, cone). We do not separate plane figures from solids. In our system the plane figures are regarded as the plane parts of the solid ones or as their projections onto a plane. We pay special attention to study the plane images of the solid figures obtained from the latter's projections onto a plane. The pupil's activity on the analysis of geometrical drawings is very important. However this activity must be carried out in the context of solving various geometrical problems and exercises. So our principal method of teaching geometry is to enable the pupils to be active learners in solving geometrical problems by using construction and analysis of the appropriate geometrical drawings.

Now we come to the point of using computer technologies in teaching geometry in school. We proceed from the premise that computer is used as a means for the more comprehensive and deeper understanding of geometry. From the numerous possibilities of computer use in school mathematics, and in geometry in particular, we consider those which have relevance to our teaching goals.

The main idea is to use computer as a means of studying space geometrical figures on the basis of their plane images by using the process of dialogue.

In many cases computer is the only means of providing the visualization principle and for searching an answer to unclear questions. Thus we deal with computer modelling of geometrical space configurations providing the following facilities:

- to study the model behaviour taking into consideration its characteristics and variations. The direction of projecting and the position of the figure in relation to the projection plane may be considered as such characteristics in particular;
- to separate parts of the whole image and to process the separated parts according to the data given;
- to separate plane figures and to study them and their relationships with the original;
- to make geometrical constructions on the image according to the problem data, to transfer the given configuration into the one required;
- to carry out the analysis and control of the results of the pupil's activity.

We achieve these facilities by working out the computer teaching system in geometry.

Computer Teaching System in Geometry

Computer teaching system is defined as an integrated computer software to provide the following functions:

- to choose a geometrical problem and the computer means of its solution;
- to get computer help;
- to input the results of the pupil's activity and to make their computer analysis and control.

The main function of the computer teaching system is to organize the individual activity of the pupils during the process of solving geometrical problems. Let us consider in detail the structure of the computer teaching system.

The choice of the problem is provided by using a computer data base consisting of the problems of some kind. This choice can be done by the pupil or by the teacher depending on the teaching process. Every problem corresponds to the file of the computer means of the solution. However from the practical point of view, we separate the main types of the problems so that every problem of one type corresponds to one and the same file of the computer means of its solution. The main types of the problems are as follows:

1. The problems of determination of the relative positions of the points, straight lines and planes;
2. Construction of the geometrical figures with the given properties;
3. Determination of the relative positions of the two space polygons and finding their common parts;
4. Construction of the geometrical proofs.

The computer means of the solutions for the problems of types 1-3 have common base and differ only in some details. The main computer means of the solutions for the problems of types 1-3 are as follows:

1. To draw
2. To delete
3. To mark

} the point, straight line segment, axis, straight line

To demonstrate how we can use these means of the solutions, let us consider the following example.

<u>Example 1</u>. *A cube is given. It is necessary to combine three of its vertices so as to obtain an equilateral triangle. Consider all possible cases (Fig. 1).*

Here, we are dealing with the computer image of a cube. The pupil's activity goes on in the following manner. First of all, for him it is necessary to mark those of the cube vertices which are the ones of the equilateral triangle. He uses the computer means of marking point by choosing the appropriate colours. Then he connects these vertices by straight line segments using the means of the solutions given.

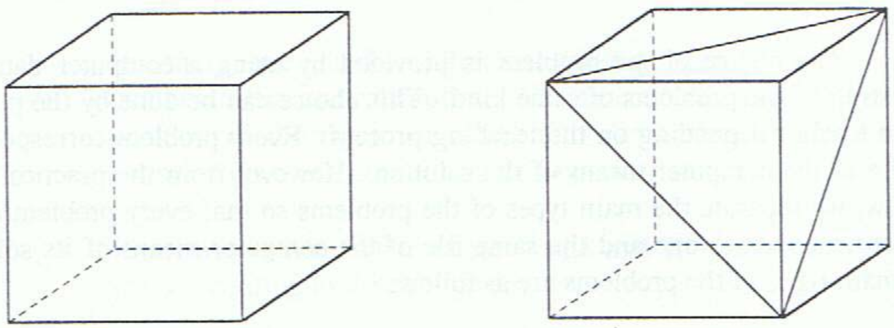

Figure 1: Cube and equilateral triangle

Example 2. A cube is given. It is necessary to combine some of its vertices so that the space polygon like the one in Figure 2 is obtained.

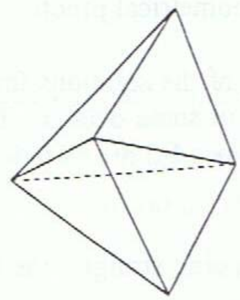

Figure 2: Space polygon with five vertices

The pupil's activity is similar to the one in Example 1. For the pupil, it is necessary to choose five of the cube vertices so that they are the required ones. For this purpose he uses the means of marking point. After that he combines the marked vertices in pairs so that the required polygon is obtained (Fig. 3). For choosing the computer means of the solution the pupil uses the menu presented on the display and operates with the mouse. The input of the results of the pupil's activity has been provided by the menu option "Input solution". We point out that the possibility to cancel the previous operation has been allowed by using the menu option "Cancel". The basis for working out the computer means of the solution is the simple graphical editors which can be worked out by using the standard procedures.

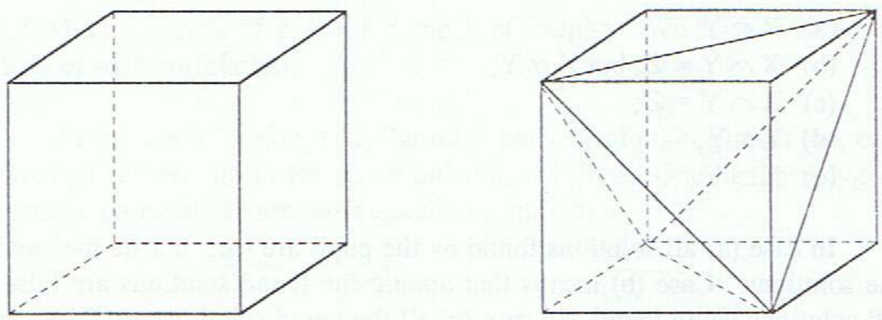

Figure 3: Cube and space polygon with five vertices

As for the geometrical problems of type 4, they are a special case. The computer means of the solution of these problems are defined as the system of geometrical statements each one of which can be true as well as false. This system has been constructed on the model of a tree (a special kind of graph) and provides the possibility of generating the logical chain which represents the problem solution. Unfortunately working out such kind of the computer means of the solution is very complicated and we are just at the starting point of this work at the moment.

Another aspect of fundamental importance is the system of computer analysis and control of the pupil's activity on solving geometrical problems. While working out this system we are faced with the following problem: what forms the mathematical grounds of the system of the computer analysis and control of the results of the pupil's activity? For some time the solution of this problem has been incomplete. However, we are now able to answer this question with confidence.

Mathematical Grounds of Computer Teaching System in Geometry

The mathematical grounds of the system of computer analysis and control of the results of the pupil's activity form the problems of pattern recognition, namely, the ones of correlation of the two sets X and Y represented as some data structures, the set Y corresponding to the problem solution and the set X indicating the results of the pupil's activity. The following cases are available:

(a) $X \subset Y$;
(b) $X \cap Y \neq \varnothing$, but $X \not\subset Y$;
(c) $X \cap Y = \varnothing$;
(d) $X = Y$;
(e) $X \supset Y$.

In case (a) all solutions found by the pupil are true, but he has not found all the solutions. Case (b) means that among the found solutions are false ones, not all solutions being found. In case (c) all the found solutions are false. In case (d) all true solutions are found, there being no extra solutions. Case (e) means that all true solutions are found, there being extra solutions among the found ones.

The problem: what data structures are the most suitable ones for the representation of the sets X and Y has been open for some time. We are not able to give the complete answer to this question. The problem is extremely complicated. But in some cases, especially in those which are related to studying plane images of the space polygons, the structures usually used for graph representation on the computer turn out to be very fruitful. It is well known that the plane image of the space polygon has been represented by the finite system consisting of the points and straight line segments. Consequently, we may consider this image as a finite graph. Let us illustrate the whole idea on Examples 1 and 2 considered above. In the case of Example 1, every equilateral which represents the problem solution corresponds to the graph consisting of three nodes and three edges (Fig. 4).

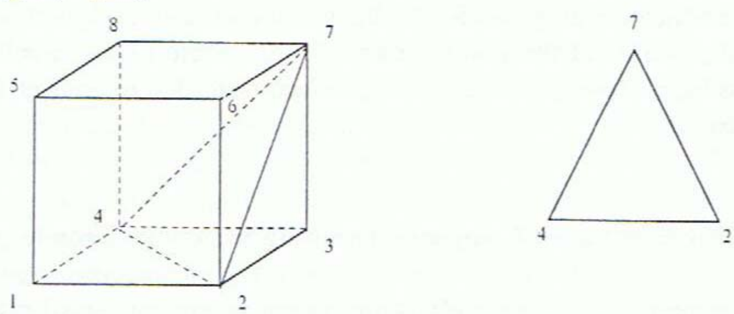

Figure 4: Cube and graph corresponding to the problem solution

In the case of Example 1 the set Y consists of eight graphs each one of which represents the problem solution. The set X must be generated as a result of a pupil's activity. So we deal with the problem of correlation of the two sets each one of which consists of a finite number of graphs. We have worked out the methods of such correlation.

In the case of Example 2, the set Y consists of eight graphs too, each one of which corresponding to the space polygon required. One of the polygons and the corresponding graph are represented in Figure 5.

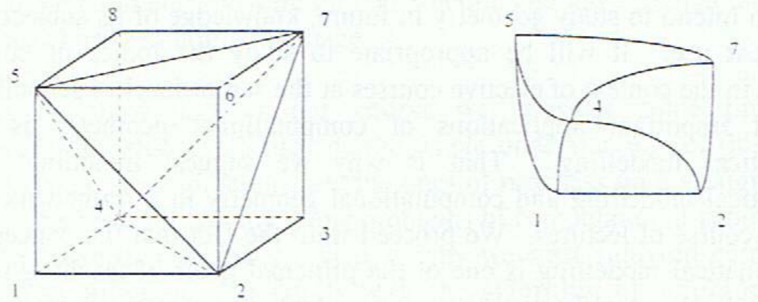

Figure 5: Space polygon and the corresponding graph

These examples show that the computer teaching systems in geometry have been constructed actually on principles of computer expert teaching systems. Their application to teaching geometry in school in association with the new approaches to education enables the teaching process to be useful, individualised and creative practically for every pupil in a classroom.

Geometry for Advanced Pupils

In every school there are pupils who go on to study mathematics at the university. Geometry training of such pupils must be given and should take into account the new trends in geometry. Special emphasis must be given to the role of computer technologies in modern geometry. It is well known that computer increasingly becomes the means of scientific research in mathematics and in geometry in particular.

The future development of geometry will be determined more and more by using computational methods and the methods of discrete mathematics. Of course, it is difficult to say at the moment what will geometry be in the 21^{st} century. However it is evident that a special place will belong to the computer.

The new trends in geometry will impact on school mathematics education. It is known that computer technologies in geometry brought into existence the new science area - computational geometry. We consider this area as one of the most hopeful in modern geometry. From this point of view we think that computational geometry is worth including in school geometry. Of course it is unnecessary for every pupil to study computational geometry, but for those of them who intend to study geometry in future, knowledge of its subject matter will be of great use. It will be appropriate to study the topics of computational geometry in the context of elective courses at the senior level of schooling. One of the most important applications of computational geometry is related to mathematical modelling. That is why we suggest including grounds of mathematical modelling and computational geometry in a framework of one and the same course of lectures. We proceed from the fact that the concept evolution of mathematical modelling is one of the principal goals of teaching mathematics in school. We familiarize the pupils with mathematical modelling using probability processes models as base. So in our course of lectures there are three directions of modern mathematics, namely, mathematical modelling, probability theory and computational geometry. In our opinion, their connection works well. We consider this approach as one of the advantages of our course. It consists of three parts. The first part is devoted to the grounds of combinatorics and a graph theory. From the broad range of modern combinatorics we naturally consider its foundations: arrangements, rearrangements, combinations and their properties. In studying the grounds of a graph theory, we pay special attention to the ways of a graph representation on computer and the problems of geometrical search on graphs.

The second part of our course deals with the grounds of a probability theory. The special feature of our teaching approach is the computer modelling of the probability processes. This approach provides the opportunity of carrying out computer experiments in support of or against the probability hypothesis. We introduce the classical definition of probability only when the experiments carried out support the tolerance of the ratio of the number of the successful trials to the one of all trials. The computer modelling of the probability processes forms the basis for the computer generation of random numbers. The special place in this part of our course is the problems of geometrical probabilities. We attach the concept of geometrical probability by the consideration of the special group of problems on studying probability processes where the classical concept of probability cannot be used. In particular, the problems of such kind are: needle of Buffon, throwing a point into a circle, throwing a coin onto a plane and some others. First of all we construct the computer models of these processes. The computational experiments on these models indicate the tolerance of the ratio of the number of the successful trials to the one of all trials as well as in the classical case. By these experiments we come to the understanding that this ratio is equal to one of the measures of the two appropriate geometrical figures. This discovery is very important and leads us to the definition of geometrical probability. We have worked out computer software for construction and have studied these models using the Pascal programming language.

In the third part of our course we consider some problems of computational geometry which provide the whole understanding of this scientific area. In particular, such problems are the ones of nearness on a straight line and on a plane. The basis for solving many problems of computational geometry is the method of quicksorting an array. That is why we start studying computational geometry on familiarizing the pupils with the algorithm of quicksorting. We complete this part by studying some applications of computational geometry to mathematical models in system of real time, catastrophe theory and biology.

Conclusions

1. The role of computer technologies in geometry research is constantly growing. This fact brings into existence the new scientific areas in geometry.

2. The new trends in geometry impact on geometry education in schools as well as in universities.

3. One of the possible applications of the computer in a framework of a compulsory geometry course in school is the computer teaching systems providing analysis and control of the pupil's activity while solving geometrical problems.

4. Studying grounds of computational geometry is worth including in the system of geometry training advanced pupils.

References

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